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Published in:
Proceedings of the Sixth Congress of the European Society for Research in Mathematics Education

Publication date:
2010

Document Version
Publisher's PDF, also known as Version of record

CERME 6 – PLENARY 2
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Summary. Based on an analysis of mathematics education research as an academic field and on current social, political and economic transformations in many European countries, I would argue for the need to rethink and enlarge definitions and views of mathematics education as a scientific field of study in order to provide better understandings and alternatives for practice in the teaching and learning of mathematics today. I will explore the notion of the "network of mathematics education practices" as a complex, multi-layered space of social practice where the meanings of the teaching and learning of mathematics are constituted. I will illustrate the potentiality of this notion to envision possible research paths in the field. I will illustrate these with the research that my colleagues and I have been carrying on multicultural classrooms in Denmark; as well as will offer examples of other research studies in Europe and other parts of the world where I see that the discipline is gaining newer insights that could allow attending to the social changes and challenges of the 21st century.

MATHEMATICS EDUCATION AS A NETWORK OF SOCIAL PRACTICES

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As academic fields advance, reflexivity on its own results and processes becomes a centre of attention and of disciplined inquiry. The growing amount of published papers and conference activities considering mathematics education, its theories, methods and results exemplify the need researchers have to make sense of the practice in which they are involved. Such type of reflexivity has always been a central part of my interest, probably due to the fact that my background in the social sciences has led me to constantly formulate questions about the type of insights on educational practices that mathematics education research offers in relation to the realities of schools and mathematics classrooms. Developing awareness on the research perspective that I adopt has, therefore, been as central to me as generating particular understandings and interpretations of the practices of teaching and learning in mathematics classrooms.

In this paper I focus on the issue of how to conceive of mathematics education as a field of research. This implies, on the one hand, examining definitions of the field as they appear in existing literature, and, on the other hand, articulating alternative views and languages to talk about the field. My intention is to provide a ground for discussing the research practices in which we engage and to which we devote a great deal of our effort and commitment. In my examination of this issue, I will contend that in the historical development of what we may identify as the field of mathematics education research, particular dominant definitions about the field of educational practices of mathematics teaching and learning have emerged. Such definitions of the educational practices have defined what the legitimate objects of study of the field of research are, and with that encompassing theories and methodologies to research the field of educational practices. As research advances, however, the definition of the field of research emerging from research practices is being pushed to its limits. I argue that the time has come to open possibilities of defining both research practices and educational practices in a way that allows tackling in serious, rigorous and systematic ways the social, cultural and political complexity of mathematics education in our contemporary societies. Opening the scope of the field does not represent a threat to the identity of the field, but rather an opportunity to engage with the enormous challenges that mathematics education practices pose to all their participants.

I start by a conceptual clarification of the language that I choose to address this issue, which entails a presentation of the underpinning ideas of my theoretical perspective. I clarify the notions of mathematics education as a field of educational practices and as a field of research practices. The distinction is useful in addressing the way in which these realms constitute each other, and of how different meanings have been ascribed

to them particularly from the second half of the 20th century when the international field of mathematics education research has been more visible and identifiable. I then move to argue that dominant definitions of the field of research and its corresponding views of its object of study are insufficient in tackling in a comprehensive manner the impact of larger contextual factors on the teaching and learning of mathematics. While research results continue to point to the influence of the "context" on actual possibilities to an effective improvement of the teaching and learning of mathematics, the field of research misses the development of scientific strategies to deal with both the understanding of those influences and the devising of strategies to deal with them in practice. As a response to this shortcoming, I play with the idea of defining mathematics education as a field of research which studies the complexities of the *network of mathematics education practices*. I define three different types of research moves or strategies that are necessary to deepening the understanding of the practices of teaching and learning of mathematics. I finalize by exemplifying these research moves with projects carried out by a growing number of mathematics educators around the world.

ANALYSING THE FIELD THROUGH ITS DISCOURSES

The increasing attention given to reflexivity in mathematics education research invites to discussions of how and why theories, methods and discourses in research are simultaneously constructed and get reproduced. In his paper during the ICMI study on what is mathematics education and what are its results, Ernest (1998) had identified the need for mathematics education research to address not only the primary objects of the field (the practices of teaching and learning mathematics), but also the secondary objects of the field (i.e., the products and processes of research practices). The growing emphasis on the effects of language and its connection to practices within the social sciences —known as the social turn— has influenced the way mathematics education researchers think about the field. Thus, it appears increasingly important to pay attention to the discourses that mathematics education research constructs about itself and the contributions and limitations of these constructions. By discourses here I understand the ways of naming and phrasing the ideas, values and norms that emerge from the constant and complex interactions among human beings while engaged in social practices. Researchers in academic fields construct particular discourses about their objects of study and their overall activity. Such discourses constitute systems of reason that regulate what is possible to think and do in a given field (Popkewitz, 2004). Thus, discourses generate both a space of possibilities as well as of limitations of what we can imagine as alternatives to existing orders.

Mathematics education as a field of research is not an exception. As researchers engage in studying the field, they not only define what characterizes legitimate practices of mathematics education. They also define the ways in which it is valid and legitimate to research those practices. I have elsewhere engaged in examinations of

the discourses generated in and by the field of mathematics education research, such as the idea of mathematics education being "powerful" (Christensen et al., 2008), the conceptions of students as mathematics learners (Valero, 2004a), and the concept of learners' identity in mathematics (Stentoft & Valero, in press). In this paper I turn to the discourses of the field about itself. My analysis is based on a study of a variety of texts addressing mathematics education research as a field of study, such as, for example, the work of Jeremy Kilpatrick (e.g., 1992, 2006, 2008; Silver & Kilpatrick, 1994), books addressing the issue (e.g., Menghini, Furinghetti, Giacardi, & Arzarello, 2008; Sierpinska & Kilpatrick, 1998) and recent handbooks (e.g., English, 2008; Lester, 2007). Drawing on elements of critical discourse analysis (Fairclough, 1995), I focus on the dominant ways of talking that emerge from the texts as they address what mathematics education practices and mathematics education research are about. The references in my analysis serve as illustrations of the characteristics of the discourses that I am identifying.

EXAMINING "MATHEMATICS EDUCATION"

The use of the term "mathematics education" in English is ambiguous. Among others, Ernest (1998, p. 72) has argued that the term refers to "both a practice (or rather a set of practices) and a field of knowledge". The term names the set of practices of mathematical teaching and learning, carried out mainly by practicing teachers and students, in a variety of formal and informal contexts, and where mathematical thinking and communication occurs. The term also refers to the set of practices, carried out mainly by researchers hired at colleges of education and universities, that study teaching and learning practices. A first thing to notice about the two meanings is that each one of them is addressing a field of practice. The former refers to the field of educational practices; the latter refers to the field of research practices. As fields of practice, each one of them has particular embodied, routinized activities, artifacts, ideas, values and forms of communication. They are distinct practices, though with intersections of practitioners (most often than not, researchers are themselves teachers and teachers are also researchers), interests, concerns and discourses. However, the two fields of practice are not identical. It is not my intention to go deeper into the characterization of these two fields of practice here. Suffice to say that their separateness or connection is a matter of concern for many practitioners located in each one of the fields (e.g., Ruthven & Goodchild, 2008; Sfard, 2005).

My intention with distinguishing the two fields here has to do with the relationship between the two, not in terms of how the field of research practice should illuminate and improve the field of educational practice; but rather in terms of how the definitions constructed for each of them are mutually constitutive. Let me explain, starting with a basic assumption. A theoretical perspective and an object of study are mutually constituted. It is not possible to talk about an object of study without a set of assumptions and language that recognizes and phrases a happening or a social event,

and makes it focus of attention. If this is the case, then we can think about the relationship between what is taken to be mathematics education as a field of educational practice and mathematics education as a field of research practice. My contention here is that through the development of the field of research practice, definitions of the field of educational practice have emerged.

Looking back at the history of the field of research practice through a general study of the different trends that have emerged in literature, as well as an examination of texts addressing the history of mathematics education research, there seems to emerge a common narrative about the origins of research. The interest of mathematicians and educators engaged in the teaching of the subject at different levels, particularly in relationship to teacher education, was a seed for paying systematic attention to mathematics in a learning and teaching environment (Kilpatrick, 2006). "The problems of practice" that is, the set of concerns for the predicaments of teachers' instruction and students' learning of mathematical topics, as formulated by Silver and Herbst (2007), have become the cornerstone of the research endeavor. The problems of practice have become the natural object of study of the field of research. They have also determined the ultimate goal of research, which is contributing to the improvement of practice. Many people defend these ideas as the essence of mathematics education research; the ideas are a central part of how many researchers define the object and aims of study (e.g., Hart, 1998). These ideas are seen by many in opposition to the idea that mathematics education research is growing as an academic field in itself, with a theoretical and methodological development that not always connects so closely with teaching and learning practices. There are also many scholars who acknowledge and actually try to understand not only the findings, but also the theoretical, methodological constructions of the field (e.g., Silver & Herbst, 2007). Of course, this debate is also fuelled by different agendas outside the field of study and the field of practice of mathematics education, such as the growing political demand for accountability of research funds and the focus on educational research to be the basis for evidence-based practice.

Independently on which side personal intentions and commitments are, two points are evident here. First, there is nothing "natural" in the definitions given to the field of research practice. The discursive construction of the object of study and the aims of research in the field correspond with the practices of researchers both in national and international communities. We actually need to denaturalize what seems to be taken for granted in the way we researchers, collectively and as individuals, talk about the field and engage with the field. Following from this, the second point is that definitions of the field of study entail definitions of the educational practices that research studies. This implies that it is not possible to assume complete independence between the social practices of teaching and learning of mathematics, from the social practices of researching them. The discourses of the field of study construe frameworks for thinking, conceiving and therefore actually engaging in the

educational practices (Popkewitz, 2004). The fields are distinct but discursively related.

Digging deeper into how the educational practices are being defined by the research practices, it is evident that definitions are historical and also situated in particular geographical settings. They are also contingent upon theories adopted to account for the problems of practice. A proper account of the complexity of the definitions exceeds the scope of this paper. Nevertheless, I will point to some salient features of the way research has been defined in general international terms. Although for many researchers the history of mathematics education research is short —in relation to the history of, say, mathematics— it is possible to find shifts in the ways of phrasing the focus of both educational and research practices. Looking at the 100-year long history of the International Commission of Mathematical Instruction (ICMI) as one international organization that has had an important role to play in promoting mathematics education research, the initial focus of the meetings, discussions and concerns of interest in the educational practices was the mathematical content. In what Bass (2008) has named ICMI's "Klein Era", at the beginning of the 20th century, attention was paid to issues of content and little distinction existed in fact between the gatherings of ICMI and the general meetings of the International Mathematical Union, except for the fact that the mathematical topics addressed in ICMI were more elemental mathematics. Such observation resonates with Kilpatrick's assertion that the work of the first mathematics educators at the end of the 18th and beginning of the 19th centuries had a strong focus on the mathematical contents, although few other topics were present as well such as the history of mathematics and teaching experiments (Kilpatrick, 2006). A graphic representation of the field of educational/research practice in this time could look like this:

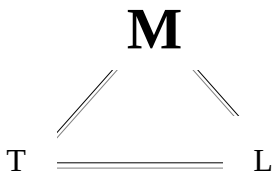

Figure 1: Mathematics at the centre of the field of practice and research

The linkage to psychology as a support discipline has been important in the construction of an empirical investigative approach towards the problems of practice. With the strengthening of parts of psychology as an experimental science and with mathematics education becoming a field in universities, mathematics education research found theoretical and methodological approaches to the inquiry of teaching and learning problems in mathematics (Lerman, 2000). The influence of the European didactic traditions have also played a major role in defining that the focus

of research is placed in the *didactic triad* constituted by the relationships between mathematics, the teacher and the student. As the 20th century advanced and more research work in the area was produced, explorations of the didactic triad had been focused on each of its elements, on the relationships among them, and on the whole complexity at stake in it. Combined with a variety of theoretical approaches to deal with the specificities of each of the elements, the didactic triad has been a basic but powerful model behind a great deal of research in the field. Saying that the didactic triad has been a model behind research in mathematics education does not intend to oversimplify and dismiss the advances of the field in understanding the complexity of the relations at the interior of the triad. There are numerous examples of particular models that have shown such complexity (e.g., Balacheff & Kaput, 1996, for the case of the role of technology in mathematics learning).

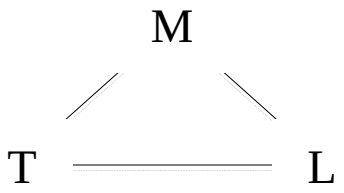

Figure 2: The didactic triad at the centre of the field of practice and research

There are several points to notice in the research and discussions about the field of research adopting this model. A first issue is the issue of the *mathematical specificity*. Mathematics education research is defined as the discipline studying "the practice of mathematics teaching and learning at all levels in (and outside) the educational system in which it is embedded" (Sierpinska & Kilpatrick, 1998, p. 29). In this field, "[…] mathematics and its specificities are inherent in the research questions from the outset. One is looking at mathematics learning and one cannot ask these questions outside of mathematics." (p. 26). Questions, problems, theories and methods not allowing for mathematical specificity tend to be considered irrelevant, and out of the scope of mathematics education research. Second, there also seems to be an underlying assumption about the *decontextualization* of the triad. The objects of research tend to be presented in terms of students' learning of concepts (and most often students' misunderstanding of them), and teachers' instruction of mathematical concepts. They are text, the content, the centre. The con-text, that surrounding accompanying and constituting the text, does not fall inside the research gaze. Therefore, except for a brief mention to the characteristics of the people involved in a study, no more grounding and information is available about the context of a given phenomenon studied. If some context is mentioned, it is not taken significantly as part of the analysis. The assumption of decontextualization goes also hand in hand with the assumption of *closure* of the didactic triad. This means that research problems are both formulated within and accounted for in the didactic triad. The

practices of teaching and learning are somehow self-contained and self-explanatory. There are plenty of examples to find in research on geometrical thinking, argumentation and proof, etc. A review of, for example, the CERME proceedings on these topics will clearly show this tendency.

As some researchers have started to consider classrooms dynamics, the classroom has appeared as a clear boundary around the triad, a clear, manageable context. One example of research contributing to the strong emergence of the classroom is the work of Cobb and collaborators during the 1990's which lead to the notion of the socio-mathematical norms (e.g., Cobb, Wood, & Yackel, 1992) which explained students' learning possibilities in terms of the continued interactions happening in the instructional practices in classrooms. In the case of Cobb and collaborators, the move from a social constructivist theory of learning to address mathematics education, to a socio-cultural theory of learning was one of the reasons for an enlarged understanding of the role of the social dynamics of the classroom in relation to individual learning. This seems to have been the case for many other researchers who started to focus on the situatedness of teaching and learning practices in classrooms and schools (e.g., Boaler, 1997).

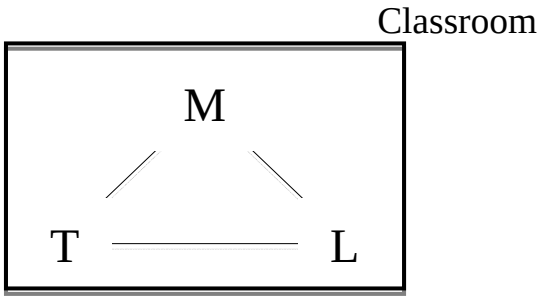

Figure 3: The didactic triad within the boundaries of the classroom

Lerman (2000, 2006) has argued that researchers in mathematics education, influenced by the language turn in the social sciences, have adopted a variety of sociological and cultural-anthropological theories for the study of the teaching and learning of mathematics. The strong social turn in the field has meant the recognition of the embeddedness of mathematical thinking, learning and teaching in larger social, cultural, economic and political structures. Research with a concern for equitable access to mathematics is an example of how such recognition has been fundamental in the generation of new research areas. In many cases, there is an attempt to stick to the formulation of problems within the didactic triad, though, from a different theoretical position. For example, part[1] of the work of Radford concerning semiotic,

[1] In few of his papers, Radford shows a broader analysis of the relation of mathematics and culture. For example, Radford and Empey (2007) present a study of social and mathematical practices outside the didactic triad. They show that "within a certain historical time period, mathematics –in its amplest sense […] accounts for the formation of new social sensibilities –both in terms of

embodied interpretations of students' mathematical thinking give a cultural dimension to the issues of the didactic triad and show a connectedness of children's thinking and school practices with other forms of practices outside schools and classrooms (Radford, 2008). In general, it is interesting to notice that, despite the adoption of theoretical frameworks that have an understanding of the social and cultural that goes beyond the limited understanding of "social" in terms of interaction among people present in interactionist theories associated with constructivism, the focus of attention of research remains being the classroom and, within it, the didactic triad.

Some other types of research have also challenged the idea that the privileged site for research is the classroom. If mathematical thinking is a social and cultural activity it happens in other social spaces different from classrooms. The classical example of this broadening is the research by Nunes and collaborators (Nunes, Carraher, & Schliemann, 1993) which opened the space for investigations of the relationships between mathematics in school and out of school. The extensive research belonging to the ethnomathematical program has also explored mathematical practices in working and everyday life settings. Already at the beginning of the 1990's Gómez, Perry and collaborators (e.g., Gómez & Perry, 1996; Perry, Valero, Castro, Gómez, & Agudelo, 1998) had studied mathematics teachers' change and professional development within the complexity of the school organization. Such trend has also been explored by Cobb and collaborators (e.g., Cobb, McClain, Silva Lamberg, & Dean, 2003) in an attempt to connect classroom communities with their immediate organizational contexts. More recently attention has been paid to the school mathematical experiences of parents in relation to the school mathematics practices of students when coming to new countries and cultures (Civil, 2007). In general, there has been a growth in research that documents the relationship between factors outside of the classroom (in the context) and the state of affairs inside the classroom, in the didactic triad.

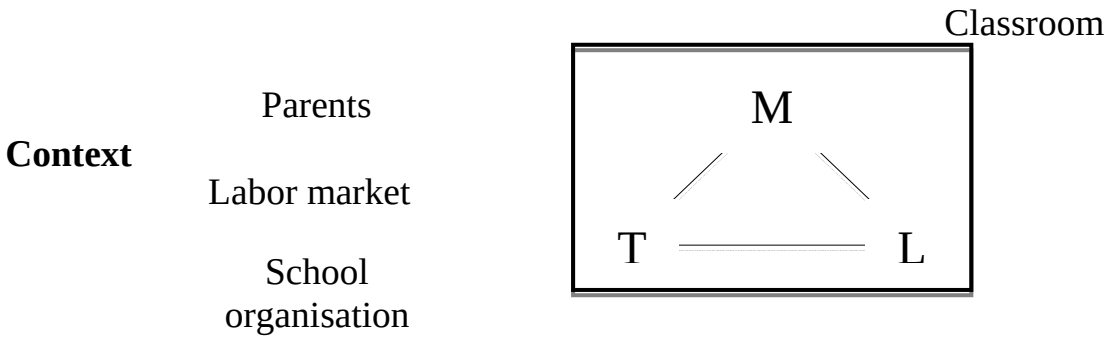

Figure 4: The didactic triad in a context

capacities to create new forms of understanding and novel forms of subjectivity" (p. 232).

In other words, the welcoming of socio-cultural theories to deal with the problems of practice has helped considering the context of those problems as a significant part of them. With such move the interpretations and understandings of the terms "mathematics" "teaching", "learning", and "thinking" are broadened and new phenomena, interactions and practices where mathematical elements are present start being included as legitimate objects of study. As an evidence of this we could look at different studies classifying the research published in different international journals and conference proceedings. All these studies assume that certain international journals actually represent the production in the field at any given time. Gómez (2000) argues that "mathematics education research production is centred mainly on cognitive problems and phenomena; that it has other minor areas of interest; and that it shows very little production on those themes related to the practices that influence somehow the teaching and learning of mathematics from the institutional or national point of view" (p. 2-3). In a review of literature focusing on how research addresses the significance of students' social class for the learning of mathematics, Chassapis (2002) also argues that little and almost insignificant attention has been paid in 30 years of research production to the issue of who are the mathematics learners and how the learners' background influences mathematical learning. This lack of attention contributes to a lack of comprehension about the social, political and cultural complexity of mathematics education and the factors involved in it. Lerman, Tsatsaroni and Xu (2006; 2002) have also produced an overview of the theories used in mathematics education research in the period 1990-2001. Their data shows that although socio-cultural theories of different types had been more used in the field, the majority of theories used in published papers are traditional psychological and mathematical theories focusing on the learners, the mathematics and the teachers. Skovsmose and Valero (2008) have also classified publications with the purpose of showing how the field gives different meanings to the term "democratic access to powerful mathematical ideas". The concentration of research on mathematical and psychological interpretations, focusing on the study of classroom practices led them to conclude that "it is highly problematic that dominant research trends in mathematics education operate within a limited scope of the space of investigating democratic access to powerful mathematical ideas. Such a paradigmatic limitation effectively obstructs the possibilities for mathematics education to face the paradoxes of the informational society". Time has passed and, as Lerman and collaborators show, the adoption of socio-cultural theories enlarges and thereby a sensitivity to define research objects outside the didactic triad emerges. However, the majority of research published defines problems that deal with the central elements of the didactic triad, and from theoretical perspectives focused on mathematical cognition. A recent overview for the papers published in ESM, JRME, MERJ, FLM, ZDM and PME proceedings during the year of 2007[2] confirms the previous findings: 25% of

[2] I thank Alexandre Pais for his support doing this overview.

papers choose as a focus a mathematical notion in learning or teaching; 29% of the papers address issues of teachers' dealing with mathematical contents and 31% choose the learners' understanding or thinking of mathematical notions. The issue remaining is how does the field of research practice address the complexity of the field of educational practice beyond the didactic triad?

OPENING UP THE CONTEXT OF THE DIDACTIC TRIAD

Although the research gaze of the field of research practices seems to be enlarged, still many researchers express a concern with the issue of dealing with the "context". Let us see at this in a more detailed way. In the first place it is important to discuss the notion of context and how the field of research defines and addresses it. In the section above I shortly defined context as the surroundings of an object –the "con" accompanying a "text". As I argued before, research approaches focusing on the didactic triad tend to ignore context, since the focus of research is the "text". In the type of research focusing on learning and thinking mathematically within the didactic triad, some understandings of "context" are present, although in the form of the context of the mathematical contents, problems or ideas that students and/or teachers deal with. This is what Wedege (1999, p. 206) calls the *task-context*.

I also argued that socio-cultural theories in mathematics education have opened for considerations of the factors that affect a classroom situation. A *situation-context*, following Wedege's formulations above has been evident in research literature, i.e., in research addressing the immediate context of teaching and learning in the classroom. But I also argued that context can be much more than the walls of the classroom. Concerning the conceptualizations of the notion of context in socio-cultural theories, Abreu (2000) has discussed how different socio-cultural theoretical trends conceptualize context, and which implications such conceptualizations have for the study of mathematical thinking and learning. On the one hand, one can consider the *micro-social and cultural contexts* of mathematics teaching and learning by focusing on "the immediate interactional setting where face-to-face interactions take place" (p. 2). On the other hand, one could focus on the *macro-social and cultural contexts* which are the "non-immediate interactional settings loosely defined by other authors as 'the broader socio-cultural systems' […] or 'broader sociocultural milieu'" (p. 2), which frame mathematical activity in any particular interactional setting. The interesting research endeavor, however, is how theories connect micro and macro contexts in a search for relationships between how individuals make sense of mathematical ideas in the complex field of activity within larger symbolic systems. For Abreu, the issue of the micro-macro relationship is not only a matter of how particular interactions with certain cultural tools mediate thinking, but also of how social valorizations of knowledge mediate individual positioning towards that

knowledge in the creation of personal identities[3]. From these perspectives, context is not just like the "the bowl that contains the soup" or the "surroundings of a text", but rather a constitutive element of the text itself. Text and context are dynamic; and they are dialectically constituted (McDermott, 1996).

In the discussion by Abreu (2000), the distinction between micro and macro context opens up for a reflection on where, on the continuum between agent and structure, mathematics education research tends to focus its research gaze. If mathematics education research is seen as a social/human field of study, it cannot escape this reflection. The classical micro-macro debate in sociology addresses the issue of whether the social world is to be understood by studying individual and their interactions or by studying social structures. Each social discipline delimits the scope of the "social" in its objects of study in particular ways. Some types of areas refer to the "social" as a broad, all-embracing functioning of human action in whole cultures and civilizations (e.g., Beck, Giddens, & Lesh, 1994). Other kinds of sociological viewpoints related to disciplines such as psychology or economics, have defined the "social" as the realm of interaction among individuals. Mathematics education researchers, in the study of the social and human phenomena of mathematical thinking, learning, teaching and education, have taken a stance in this discussion implicitly (more often than not). Mathematics education research, as characterized previously with a focus on the didactic triad, has tended to focus so much in individual *mathematical* thinking, reasoning and cognition that the "social" dimension was almost non-existing. One example of this could be mathematics education as seen from a radical constructivist perspective centered on individual reorganizations of mathematical ideas. Social constructivism and related views of learning opened for a social dimension in terms of inter-personal interactions. It is only with certain recontextualizations of socio-cultural theories that the understandings of the social move beyond the individual and inter-individual level and, as Abreu says, push for the need of establishing a connection between micro and macro levels of the social. Nevertheless, studies in mathematics education from socio-cultural perspectives have also tended to focus on micro-contexts, probably because the dominance of discourses of the field of study with a centre on the didactic triad, and with a closeness to the "problems of practice" define the legitimate problem field in terms of micro-interactions and micro-contexts. The interesting question that emerges here is whether focusing on objects and problems in a micro-sociological level is the only possibility for mathematics education research. I will return to this point.

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[3] The research of Guida d'Abreu offers an interesting example of the different notions of context put in operation in research on mathematical practices. From her earlier research on Brazilian sugar cane farmers to her recent work on the valorizations of mathematics among immigrant children and parents in England (Abreu, 2007), it is possible to identify the differences in theoretical perspectives concerning how to deal with the significance of context in relation to mathematical practices.

Addressing context —and with it the many factors, actors, meanings and discourses that are difficult to grasp at a micro-social level but that researchers know have a great influence on the micro settings that we choose to research— is a difficult matter. In systematic readings of literature, researchers point to the need of research that actually deals with both the micro-complexity and the macro-complexity of mathematics education. I present here a selection of studies from different types of research and theoretical orientations that illustrate this concern.

In the USA and dealing with the concern of how to expand massively the constructivist-inspired vision of school mathematics of the NCTM, Confrey (2000a, 2000b) argued that it was necessary to expand constructivism from the level of a learning theory operating at individual or classroom level, to the level of a system. She urged for a view of research that could go beyond the micro-findings of research:

> […R]esearch never anticipated all of the leaks in the bucket, nor did it bring strongly enough into relief the fact that the bucket is only a small part of a large system. It is undeniable that researchers identified critical issues […] Despite the importance of these results, changing any one of them alone was proving insufficient to fix the problems of mathematics and science. […] All of these changes require one to look more broadly, beyond the restricted focus of a research study. All of them require us to move beyond the level of the classroom, a move that occurs only rarely in educational research. (Confrey, 2000a, pp. 88-89)

An examination of research and development initiatives in the USA to bring democratic access of students to the goods associated with high achievement in mathematics, Rousseau and Tate (2008, p. 315) conclude:

> The factors influencing democratic access in mathematics education are complex. If we look strictly at events as they occur in the classroom, without consideration of the complex forces that helped to shape those learning conditions, our understanding is only partial [and] the solutions to the problem [are] ineffectual. We must seek to reach a fuller understanding of the complex issues that shape access and opportunity to learn in mathematics so that, in turn, we can develop more effective strategies to ensure access and opportunity for all students.

In the area of teacher education, studies on the professional development of mathematics teachers and on their learning have argued and shown the importance of broadening the understanding of what is at stake when professional teachers do their work and learn. Krainer has pointed to this systematically since the end of the 1990's. More recently (2007, p. 2), he writes:

> It is important to take into account that teachers' learning is a complex process and is to a large extent influenced by personal, social, organisational, cultural and political factors.

Acknowledging the multiple influences in teachers' learning, the third volume of the International Handbook of Mathematics Teacher Education (Krainer & Wood, 2007) is organized around chapter addressing teachers' professional learning at individual,

team, community and network levels. The book as a whole illustrates research that moves beyond individual teachers and classrooms.

The examples above represent few key studies of people who, in different research areas and during the last 10 years, have argued for a need to expand the scope of research of the field. If mathematics education research ought to tackle systematically not only the micro-contexts of mathematical teaching, learning and thinking, but also its macro-contexts and the relationship between the two types of contexts, it is evident that definitions of the field of study centered on the didactic triad and recognizing the existence of a context are not enough. I will now engage in exploring a proposal of what the field of research practices, and therefore, the field of educational practices could be thought of.

MATHEMATICS EDUCATION AS A NETWORK OF SOCIAL PRACTICES

Our understandings of mathematics education as a field of research practices need to be enlarged, and with that our understandings of the practices that are the objects of study of the research field. This idea has always been part of a concern that has emerged from my research experience in Colombia as part of the team of researchers called “una empresa docente” at the Universidad de los Andes in Bogotá, later on as part of my doctoral studies at the Danish University of Education in Denmark, and now as part of the research group in mathematics and science education at Aalborg University in Denmark.

This idea has been developing since 1999 when, in the exploration of the relationship between mathematics education and democracy, I wrote:

> First, the justifications to connect mathematics education to democracy are not only found in the mathematical content, but also and mainly in the social and political factors that constitute the learning and teaching relationships in the classroom, in the school and in society. Second, and as a consequence of the latter, it is necessary to study the context of the practices and its components. By doing so, we could gain a better understanding of what mathematics education for democracy means in other instances where the social relationships that constitute and shape mathematics teaching and learning are built. Thus, a definition of the social practices of mathematics education should include not only all the institutionalized relationships among teachers, students and mathematics at the different levels of schooling, inside and outside the educational system, but also the activity of policy makers that at a national level deal with the design of curricular guidelines for the teaching of mathematics […]; the activity of writing mathematics textbooks […]; the complex relationships that configure the teaching of mathematics within the organizational structure of educational institutions […]; the spaces of teacher education both in its initial […] and further stages […]; as well as the configuration processes of social conceptions about the role of mathematics education in society […]. All these practices together should be potential and legitimate objects of study if we aim

> at understanding and, at the same time, promoting a mathematical education for democracy. (Valero, 1999, p. 21)

My initial concern for the relationship between mathematics education and democracy within the framework of critical mathematics education proposed, among others, by Skovsmose (Skovsmose, 1994) has evolved to become a general concern for developing a socio-political approach to mathematics education. As I have argued elsewhere (Valero, 2004b, 2007), such an approach views mathematics education as social practices where power relationships among the participants in and the discourses emerging from the practices are an important constitutive dimension. In contrast to a socio-cultural perspective to read mathematics education where the issue of power is not dealt with explicitly or is hidden in the valorization of practices and meanings within semiotic systems, a socio-political approach privileges power.

The concept of the *network mathematics education practices* has been under construction for a while and it has been named slightly different in my different writings (Valero, 2002, 2007, 2009). This paper has been an opportunity more to clarify the views, assumptions and analysis behind such notion. More than a finished concept, I see the concept as being still under construction. But what does this notion refer to?

In the first place, if mathematics education practices are to be defined beyond the didactic triad and in relation to their broad context, it is necessary to define "mathematics education" not only in terms of the agents and phenomena strictly related to mathematical thinking, teaching and learning, but also in terms of the series of social practices that contribute giving meaning to the activity of people when thinking, learning and teaching mathematics, as well as when engaging in situation where mathematical elements are present. Thus, the *meaning* of mathematical thinking, teaching and learning is not exclusively related to the particular meaning of the mathematical content and concepts in learning and/or teaching situations[4]. Meaning is also related to the significance given to the mathematical rationality within a diverse series of social practices constituting educational practices in a given historical time. Behind this idea there is the clear recognition that what we understand by mathematics is far from being a unified body of knowledge determined by the practices of professional mathematicians, but rather a series of "knowledges" and "language games" bounded to a diversity of practices, all of which have a family resemblance. The recent work of Knijnik (2008) in ethnomathematics is useful here to discuss the issue of meaning and diversity of mathematics in relation to social practices. The work of Sfard (2009) in identifying the irresistible pervasiveness of *numberese*, the numerical discourses in our societies, is useful in understanding how

[4] Skovsmose (2005) has pointed to this idea in relation to the sense that students make of mathematical ideas. For him meaning is constructed in relation to the students' foregrounds and the role that mathematics plays in how students perceive their future possibilities in life.

numerical discourses associated with the diversity of language games of mathematics in our society constitute ways of seeing the world. If mathematics-related language games are present in many spheres of practice, the meaning of them are also constituted in relation to those practices and their discursive elements.

Second, which is the diversity of social practices where the meanings are constituted? Mathematics education as a field of educational practice can be defined as a series of social practices, carried out by different people in different sites, where the meaning of the teaching and learning of mathematics is constituted, in particular historical conditions. Those social practices are to be found not only in the classroom where teachers and students interact around mathematical content, but also in, for example:

- family practices and parents' demands to school (mathematics)
- local community practices and their educational needs
- international or national educational policymaking practices in mathematics, which structure and regulate the forms of valid knowledge, competences and achievement levels to be attained by students and teachers in mathematics
- teacher education practices
- textbook production practices
- labour market practices and expectations on the mathematical qualifications of workers
- mathematics education research practices
- mathematics research practices
- youth culture practices
- mass media practices and the construction of public views and discourses of mathematics
- practices of international comparisons of (mathematics) achievement

Many other sites of practice could be mentioned and could be identified to be relevant at a given historical time. As an example, we could consider the role of the international comparative studies that, from the time of TIMMS in the middle of the 1990's have had a great influence in national policies, local curricular changes and teachers' work. Particular meanings of what counts as mathematics education have been put forward through the impact that results of these comparisons have had on adjusting mathematics educational policies in many countries. The PISA studies have also brought with them definitions of mathematical competency that have been incorporated in several European countries. These definitions have framed what at this historical time policy makers, teachers and researchers understand by mathematical competence. The work of Jablonka (2009) evidencing this rationality is useful in seeing how the PISA rationality has permeated many other spheres of practice in mathematics education. Whether international comparisons will keep on having such a defining role in the network of mathematics education practices in the future depends on political and economic configurations of the discourses that will rule educational thinking in the years to come. As for mathematics education it is

clear that such an element has impacted in this historical time. The future also remains uncertain.

By using the idea of a network —in contrast with the use of the concept of system— I want to convey the idea that these various sites of practice, their participants, organization, rules and discourses, are sometimes loosely and sometimes tightly coupled depending on particular historical circumstances. It is not possible to assume a particular general dynamic and development of the practices, except from the idea that many of them are implicated in the construction of the multiple meanings ascribed to mathematics education in a given time and location. In this sense, this notion is different from, for example, the vision proposed by Confrey (2000b) of a constructivist learning system.

Figure 5 is an attempt to represent the network of social practices of mathematics education, as far as my two-dimensional expertise for this kind of drawings permits to grasp the idea. The "bubbles" represent a site of practice. Notice that some bubbles are empty. With this I want to convey the idea that many practices may be considered at a given time. The connecting lines may sometimes be weaker and some times be stronger. A better representation would be to imagine a 3-D constellation of bubbles that move, become bigger or smaller, and connecting in distinct ways at different times.

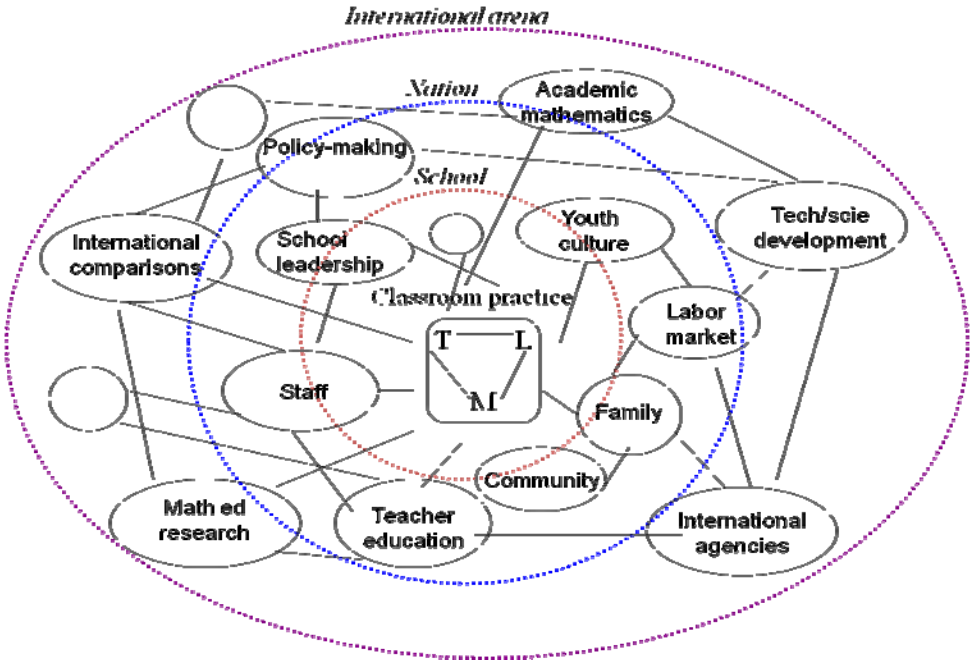

Figure 5: A representation of the "Network of mathematics education practices"

For me, defining mathematics education in terms of the network of mathematics education practices allows to evidence the cultural, social, economic, historical and political complexity of mathematics education. It also opens for envisioning a quite distinct field of research practices that, besides dealing with the objects and relationships that it has addressed until the moment, can engage in other types of research moves, with the double aim of generating deeper understandings and

interpretations of the field and of addressing the problems of practice of the multiple participants in this broader field.

If the field of research deals with the study of the field of educational practices defined in terms of the network, then three issues become evident. Firstly, the field of research and any study within it can be defined in terms of the *mathematical specificity* of it. However, the mathematical specificity of mathematics education research cannot be defined mainly in terms of the particular mathematical content, notions or competencies being addressed in the research. Rather, it has to be defined in terms of the significance of the mathematics-related practices and rationalities for the construction of the meaning of such practice, or other related practices, among its participants. When discussing research the concern of some researchers with the mathematical specificity of a given project is often expressed though questions such as: "But… would it matter if one changed the word 'mathematics' for the word 'geography' or 'history' in this project?" If we understand the mathematical specificity of mathematics education research in the broader terms proposed here, questions such as the one above will become completely irrelevant and would not be anymore a question to judge whether a research is a "proper" mathematics education research. If a research addresses in substantial ways the meaning and importance that different participants give to mathematics-related practices, or how mathematics-related rationalities that have an impact on the way mathematics education discourses are formed, then a research could be part of the field of mathematics education. In other words, the mathematical specificity of the field is related more to the social valorization that mathematics-related practices have in the dominant cultural, social and political order, and not to an explicit mathematical content or knowledge being researched. Such valorization is associated to the status of the field as a power/knowledge, which allows participants in mathematical-related practices to gain a positioning in relation to other people. That we study mathematics-related practices and their relation to the meaning of mathematics education has therefore a social and political significance, even if there is no apparent mathematical content involved.

Second, the study of any of the practices involved in the network has to acknowledge seriously *contextualization*. In contrast to the decontextualization that dominates in views of the field focusing on the didactic triad, researching the network of mathematics education practices invites to search for the intricate relationships between different sites of practice in constituting each other. The contextualization of mathematics education practices point to the contingency of practices and discourses when people engage in the task of giving meaning to mathematics-related ideas and practices in educational spheres or in any other sphere of human action[5].

[5] For an example of a study exploring the significance of contingency and complexity when researching mathematics education practices see Stentoft (2009).

Third, the view of mathematics education as a network of social practices implies that research problems do not need to be defined nor addressed within the didactic triad in a closed manner, but rather they can be formulated and tackled in the *openness* of the sites of the network. While a closed view of the field of research and practice will tend to become internalistic and provide problems and explanations within the realm of the elements involved in the didactic triad, the network of mathematics education practices highlights that the problems researchers formulate and their interpretations are always fragmented and cover only one little part of the complexity of practice.

The issue that I will engage with now is: how is it possible to do research in the "hyper-complexity" that the network of mathematics education suggests?

RESEARCH MOVES IN THE NETWORK

Whenever we do research, we perform a "move" or a strategy in the process of constructing knowledge about the objects involved in our study. It is obvious to say that these moves depend on theoretical and methodological frameworks, as well as on the traditions of the field of study. In mathematics education as a field of research practice focused mainly on the didactic triad, the most frequent research moves can be characterized as strategies addressing a very well defined research object, where the complexity of variables or factors considered is limited in order to make research projects manageable and realizable. The research move has been then a move towards an in-depth exploration of few factors and actors. The result of such move has been the production of a considerable amount of knowledge about how factors work in isolation, at the expense of how they interact together. Confrey's quotation cited above pointed precisely to this characteristic of mathematics education research. Some people call this the "fragmentation" of the field, which could be solved by striving for unification of theories. Whether this unification is possible and desirable, and actually can contribute to address the fragmentation is an issue of debate in the community. I do not think that striving for unification is neither possible nor desirable. I agree with Lerman (2006) in the argument that the apparent "fragmentation" is a very condition of the endeavor of researching social and human processes such as mathematics education, at the historical time we are living now. Rather, I would argue that fragmentation emerging from research moves that try to cover the depth of defined problems needs to be complemented by different research moves that provide needed problematization and better insight into the social and political complexity of the multiple practices of mathematics education. In what follows, I will formulate three research moves —among many others one could think of— for researching the network of mathematics education practices.

If mathematics education practices are seen as the network I proposed, the aim of the research field would be to provide insight into not only how each single node of the network operates constructing the meaning and significance of mathematics education, but also into how different nodes interconnect at particular historical

times. A research move aiming at *covering the breadth of the social practices of mathematics education* would then "slice" and define objects of study in a different manner. It would define problems in terms of the interrelationships of different nodes in the network.

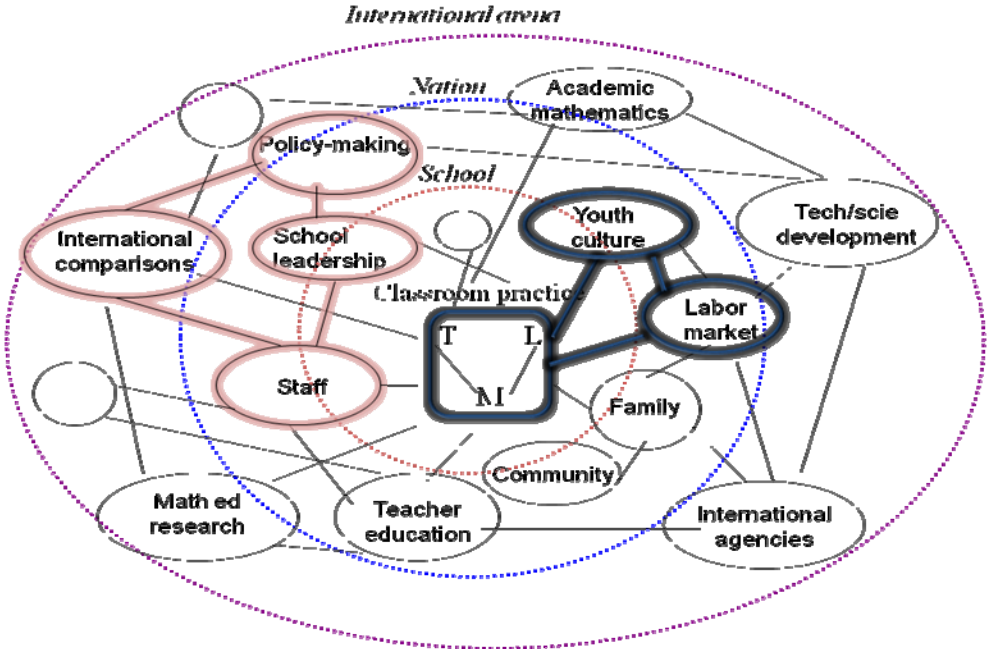

Figure 6: Defining research moves in the "Network of mathematics education practices"

The highlighted areas in the diagram above illustrate possible ways of "slicing" the network in a research move trying to gain breadth in the research. The area highlighted in the right side of the diagram would correspond to a study of, for example, how international comparisons in mathematics have affected national policy making, school leadership and demand for change to mathematics teachers in schools at the level of staff organization. The highlighted area to the right could correspond to a study on teaching and learning cultures in the classroom in relation to youth culture and demands from the labor market. The study of Zevenbergen (2005) on "Millennial" young people's numeracies at the workplace is close to such a kind of exploration of the network.

Other examples of such a research move for breadth is Martin (2000) who examines how the systematic failure of Afro-American students in the USA is constituted in a multilayered space of individuals, schools, families and communities. He shows how the mathematical identities of the students in his study can only be seen and interpreted in this multiple, interconnected levels. The research of Alrø, Skovmose and Valero (2008) argue and document the need of expanding the lenses for researching learning possibilities and conflicts in multicultural mathematics classrooms by considering the interconnectedness of at least nine different settings of practice: students' foregrounds, students' identity, teachers' perspectives of and

priorities in mathematics teaching, classroom interaction, the mathematical content, friends' priorities for participation in mathematics education, parents' expectations of mathematics education, the tools and resources available and the public discourses on diversity and education.

Another important strategy is *moving back and forth along the continuum of agency and structure* or, in other words, micro-social and macro-social units. One example of this type of move is the work of Gellert (2008), who in examining the issue of comparing and combining different theoretical frameworks, delineates a general methodology that, based on interactionist and structural theories, allows to interpret how the mathematics classroom discourses and practices are implicated in the re-construction of social in(ex)clusion. Morgan (2009) also presents a study that, within the framework of critical discourse analysis, shows how the differential discourses of mathematical ability in curricular documents and textbooks targeted towards students with different attainment levels generate differential educational possibilities for different types of students. This study illustrates that ideas and discourses of individual mathematical ability are not only produced in the classroom, but are also produced in institutionalized practices at a level of structure that goes beyond the individual participants in mathematics education practices in classrooms. These two studies exemplify research moves, with their corresponding theoretical and methodological tools, that connect the micro and the macro contexts of mathematics education.

Yet another strategy is *moving along time to find the historical constitution of the meanings of mathematics education*. Such a move evidences the contextualization of mathematics education practices in particular social configurations. Inspired on the archaeology and genealogy of practices and discourses suggested by Foucault, Knijnik and her collaborators have been recently exploring how different central ideas in the field of mathematics education have come to be created. One example is the research by Duarte (Duarte, 2008) on how the idea of the necessity and importance of connecting school mathematics and the world out of school —or the "real" world— has emerged in the particular case of Brazilian mathematics education discourses. The study digs in the history of education in Brazil and identifies the historical moment in which the conditions for the introduction of such idea took place at the beginning of the 20th century. At the same time, the process of recontextualization of the idea in relation to mathematics education is shown through an analysis of mathematics education journals and conference proceedings in recent times. Other studies (Knijnik, Wanderer, & Duarte, 2008) examine and problematize how other ideas such as the necessity of using concrete materials have become part of the dominant discourses of mathematics education.

TOWARDS THE FUTURE

Mathematics education research has grown as a field of educational research. It has expanded in terms of the amount of results produced, the diversity of theoretical approaches and the richness of the problems addressed. Mathematics-related practices in schools and in different social spheres of action also become more and more evident to different participants in those practices. Whether mathematics education research has the potentiality for addressing in significant ways those practices and generating interesting insights about them, is a matter of how far researchers —as well as practitioners— want to engage in the exploration of the social, cultural, historical, political and even economic significance of them in the construction of society.

Enlarging the scope of the field in terms of the network of mathematics education practices poses both intellectual and ethical challenges. Researching the network of mathematics education practices through, among others, the three types of research moves I suggested here demands much more collective effort, and much more sustained interdisciplinary collaboration with colleagues with expertise in other research fields. I am well aware that, given the tighter funding possibilities for mathematics education research at this moment and the increasing publication demands from university administration, constructing research agendas in this line is an ambitious task. Nevertheless I still think that more studies in this line will help the field gaining a richer insight and understanding into the functioning of mathematics education in society. Tackling the complexity of mathematical thinking, learning, teaching and rationality in our societies is definitely an intellectually sophisticated and demanding —as well as fascinating— endeavor.

It is also an ethical challenge in that an honest concern with the betterment of practices —and with the many tortuous and disenfranchising school experiences of many children around the world— demands taking political risks that go beyond the known boundaries of established disciplines and fields of research. Moving the boundaries of a research field such as mathematics education is an ethical commitment with what our work as educators and researchers has to offer to our selves, our children and the generations to come.

I hope that the complexity that suggests the network of mathematics education practices can question the very many comfortable, good and predictable research results that pullulate in the field, and open the space for a third epoch of research concerned and committed with the relationship between mathematics, education and society. As suggested by different participants in the ICMI Centenary symposium in Rome in March 2008 (i.e., Artigue, 2008; Blomhøj, 2008, p. 172; da Ponte, 2008, p. 110; Povey & Zevenbergen, 2008, pp. 285-286) as an international community we have gained awareness of the complexity of mathematics education. The European community represented in CERME can certainly contribute in that direction. It is time to do it!

ACKNOWLEDGEMENTS

I would like to thank the members of the Science and Mathematics Research Group (SMERG) at the Department of Education, Learning and Philosophy at Aalborg University, in particular Annica Andersson, Alexandre Pais, and Troels Lange, and Tamsin Meaney from Charles Sturt University, for many conversations and discussions during the preparation of this paper. I would also like to thank Margarida César, University of Lisbon, for her patience as a reactor to my presentation and paper, and for the very good comments and provocations in our preparation of the plenary session for the conference.

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